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SPURIOUS REGRESSION

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Abstract

The spurious regression phenomenon in Least Squares occurs for a wide range of Data Generating Processes, such as driftless unit roots, unit roots with drift, long memory, trend and broken-trend stationarity. Indeed, spurious regressions have played a fundamental role in the building of modern time series econometrics and have revolutionized many of the procedures used in applied macroeconomics. Spin-offs from this research range from unit-root tests to cointegration and error-correction models. This paper provides an overview of results about spurious regression, pulled from diverse sources, and explains their implications.

Keywords: Spurious Regression, Stationarity, Unit Root, Long Memory

JEL Classification: C10, C12, C13, C22.

Introduction

During the last 30 years econometric theory has undergone a revolution. In the late seventies, economists and econometricians recognized that insufficient attention was being paid to trending mechanisms and that, in fact, most macroeconomic variables were probably nonstationary. Such an appraisal gave rise to an extraordinary development that substantially modified the way empirical studies in time-series econometrics are carried out. Research in nonstationarity has advanced significantly since the early important papers, such as Granger and Newbold (1974), Davidson, Hendry, Srba, and Yeo (1978), Hendry and Mizon (1978), Plosser and Schwert (1978) Bhattacharya, Gupta, and Waymire (1983) and Phillips (1986). Nelson and Plosser (1982) asserted that many relevant U.S. macroeconomic time series were governed by a unit

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root (a random trending mechanism), based on Dickey and Fuller's (1979) Unit-root test. Several years later, Perron (1989) argued that the trending mechanism in macro variables was deterministic in nature (with some transcendent structural breaks). The debate continues between 'unit rooters' and 'deterministic trenders', though there is very general consensus as to the presence of a trending mechanism in the levels of most macroeconomic series. In the words of Durlauf and Phillips (1988):

"Traditional Analyses of Economic time series frequently rely on the assumption that the time series in question are stationary, ergodic processes [...]. However, the assumptions of the traditional theory do not provide much solace to the empirical worker. Even casual examination of such time series as GNP reveals that the series do not possess constant means".

Econometrics should work hand-in-hand with economic theory by providing it with the tools it requires to understand economic activity. The modeling of such mechanisms is thus a major goal of time series econometrics.¹ Spurious regression can be considered as having played a fundamental role in this development. To understand it, we paraphrase Granger, Hyung, and Jeon (2001), who provide an illuminating definition [Phillips (1986) showed analytically that, when regressing two independent stochastic trends, the estimates of the regression coefficient do not converge to their real value of zero.² Phillips's (1986) results are detailed in a simple case in Appendix A]:

"A spurious regression occurs when a pair of independent series, but with strong temporal properties, is found apparently to be related according to standard inference in a Least Squares regression".

Phillips (1998) presented a counterargument on the usefulness of spurious regression: trend specifications are just coordinate systems for representing behavior over time. Phillips argues that even if the series are statistically independent, when they include a trending mechanism in their DGP, they admit a regression representation (even in the absence of cointegration). This is in sharp contrast to the usual concept of spu-

rious regression. We usually conceive this phenomenon as the statistical identification of a commonality of trending mechanisms. Phillips ventures that such results—the ‘spurious’—constitute an adequate representation of the data. His main result applies for regressions among stochastically trended series on time polynomials as well as regressions among independent random walks. Phillips (1998) proves that Brownian motions can be represented by deterministic functions of time with random coefficients. Given that standardized discrete time series with a unit root (hereinafter UR) converge weakly to Brownian motion processes, it is argued that deterministic time functions may be used to model them. Such representations include polynomial trends, trend breaks as well as sinusoidal trends; it is also proved that a stochastic process can represent an arbitrary deterministic function on a particular interval, so a regression of a UR process on an independent UR process is thus also a valid representation of the data. In both cases, the t-statistics diverge at rate $T^{\frac{1}{2}}$, which is consistent since such parameterization reflects a partial—though correct—specification of the DGP. One of the most significant conclusions of Phillips concerns the long-standing debate of UR versus Trend Stationarity. To quote Phillips:

“[. . .] Our results show that such specifications [Trend stationary processes] are not, in fact, really alternatives to a UR model at all. Since the UR processes have limiting representations entirely in terms of these functions (deterministic), it is apparent that we can mistakenly ‘reject’ a UR model in favor of a trend ‘alternative’ when in fact that alternative model is nothing other than an alternate representation of the UR process itself.”

This perspective has the virtue of allowing variables with different trending mechanism (deterministic or stochastic) to be related without being limited to the somewhat restrictive case of cointegration. Phillips advances this as an appropriate approach to study stochastically unbalanced relationships such as the ones that may arise between variables such as interest rates, inflation, money stock and GDP [for further detail, see

Phillips (2003)].

To the best of our knowledge, little has been done in the way of bringing the most important works in this field together, treating them in any kind of standardized way, making connections between them, and of any real study of the profound implications for economics they might indicate. This article aims to rectify this situation.

1 Appraisal of the spurious phenomenon

Much progress has been made with Least Squares statistical inference since it was first proposed more than two centuries ago as a means of estimating the course of comets (Legendre 1805). Theoretical developments in econometrics address the non-experimental nature of economic data sets. Least Squares (LS) offers a trade-off between simplicity and powerful inference. Nevertheless, LS has certain limitations, such as potential confusion between correlation and causality, and used unwisely may produce misleading evidence. Statisticians and econometricians had been aware of the "spurious phenomenon" since Yule (1897) and Pearson (1897) [for excellent reviews of these works, see Hendry and Morgan (1995) and Aldrich (1995)]. These results led to the common expertise in the time-series field that indicated the need to differentiate potentially nonstationary series when using these to run regressions or detrending these by fitting trend lines estimated with LS. See Morgan (1990).

There are many examples of spurious regression. Some of these are commented on in Phillips (1998), where we discover the implausible relationship between 'the number of ordained ministers and the rate of alcoholism in Great Britain in the nineteenth century'; the equally 'remarkable relationship' presented in Yule (1926) concerning the 'proportion of Church of England marriages to all marriages and the mortality rate over the period 1866-1911'; the 'strange relationship' between price level and cumulative rainfall in the UK, which was advanced as a curious alternative version of quantitative theory by Hendry (1980). Plosser and Schwert (1978) presented another example of

nonsense correlation when they proposed their quantity theory of sunspots. The main argument is that the log of nominal income can be explained by way of the log of accumulative sunspots. Not only did they find statistically significant estimates, but the goodness of fit, measured with the R^2 , is quite high: 0.82.³ Granger and Newbold (1974) computed a Monte Carlo Experiment where a number of regressions, specified as equation (1), were run using simulated variables, each perfectly independent of the others.

$$y_t = \alpha + \beta x_t + u_t \quad (1)$$

where $t = 1, \dots, T$, being T the sample size. The variables x_t and y_t are independent. Under standard regularity conditions, LS delivers no evidence of a linear relationship between y and x . In particular, $\hat{\beta}$ should be statistically equal to zero. Nevertheless, Granger and Newbold's (1974) they were generated as $I(1)$ processes,⁴ usually referred to as random walks, that is, UR processes, but found estimated parameters statistically different from zero, with its associated t-ratio $t_{\hat{\beta}} = \frac{\hat{\beta}}{\hat{\sigma}_{\hat{\beta}}}$, unusually high.⁵ Phillips (1986) provided a theoretical framework that explained the causes of the phenomenon of spurious regression. In short, it is fair to say that standard LS inference can only be drawn when the variables are stationary. Even with stationary but highly persistent variables, spurious regression can occur when the standard errors used in the t-ratio are inconsistent. Ferson, Sarkissian, and Simin (2003) provide examples in financial economics. One extremely important exception to this is the case of cointegration. Even if the series are stochastically trending, when the trend is common to both series the LS regression then works particularly well in the sense that estimates converge in probability to their true value at a rate faster than T , but have a nonstandard distribution (Stock 1987).

2 Data Generating Processes

Research on spurious regression has been making use of increasingly complex Data Generating Processes (DGPs). Table (1) provides a summary of those appearing in this survey:

#	Name	Model
1	$MA(q_w)$ or $AR(p_w)$	$w_t = \sum_{i=1}^{q_w} \Theta_{iw} \epsilon_{wt-i}$ or $w_t = \sum_{i=1}^{p_w} \phi_{wi} w_{t-i} + \epsilon_{wt}$ (stationary)
2	$I(0)$	$w_t = \mu_w + u_{wt}$
3	$I(0) + br$	$w_t = \mu_w + \sum_{i=1}^{N_w} \theta_{iw} DU_{iwt} + u_{wt}$
4	TS	$w_t = \mu_w + \beta_w t + u_{wt}$
5	$TS + br$	$w_t = \mu_w + \sum_{i=1}^{N_w} \theta_{iw} DU_{iwt} + \beta_w t + \sum_{i=1}^{M_w} \gamma_{iw} DT_{iwt} + u_{wt}$
6	$I(1)$	$\Delta w_t = u_{wt}$
7	$I(1) + dr$	$\Delta w_t = \mu_w + u_{wt}$
8	$I(1) + dr + br$	$\Delta w_t = \mu_w + \sum_{i=1}^{N_w} \theta_{iw} DU_{iwt} + u_{wt}$
9	$I(k)$	$\Delta^k w_t = u_{wt}$ for $k = 2, 3, \dots$
10	$FI(d)$	$(1-L)^d w_t = u_{wt}$ for $d \in (0, \frac{1}{2})$
11	$FI(1+d)$	$\Delta w_t = u_{wt}$ with $(1-L)^d u_{wt} = \epsilon_{wt}$ for $d \in (0, \frac{3}{2})$

Table 1: The DGP's for $w_t = x_t, y_t, z_t$. *Note:* TS, br and dr stand for trend-stationarity, breaks, and drift, respectively.

where u_{wt} are independent innovations obeying Assumption 1 in Phillips (1986), ϵ_{wt} is an iid white noise with mean zero and variance σ_ϵ^2 , and DU_{iwt}, DT_{iwt} are dummy variables allowing changes in the trend's level and slope respectively, that is, $DU_{iwt} = \mathbf{1}(t > T_{b_{iw}})$ and $DT_{iwt} = (t - T_{b_{iw}})\mathbf{1}(t > T_{b_{iw}})$, where $\mathbf{1}(\cdot)$ is the indicator function, and $T_{b_{iw}}$ is the unknown date of the i^{th} break in w . We denote the break fraction as $\lambda_{iw} = (T_{b_{iw}}/T) \in (0, 1)$, where T is the sample size; $d \in (-\frac{1}{2}, \frac{3}{2})$. Only DGPs 1, 2 and 10 (for $d < 0.5$) satisfy the weak stationarity definition. The remaining DGPs generate nonstationary series.⁶ FI processes deserve further discussion; contrary to regular $ARMA(p, q)$ processes—made popular by Box and Jenkins in the 1970s—such as DGP 1, whose autocorrelation function decays at an exponential rate (short memory), $FI(d)$ processes have an autocorrelation function that decays at a hyperbolic rate (long

memory).

There are a number of empirical examples in time series in which dependence falls slowly across time. This phenomenon, known as Long Memory or Long-Range dependence, was observed in geophysical data, such as river flow data (Hurst 1951) and in climatological series (Hipel and McLeod 1978), as well as in economic time series (Adelman 1965). In three important papers (Granger 1980, Granger and Joyeux 1980, Hosking 1981), the authors extended these processes to provide more flexible low-frequency or Long Memory behavior by considering $I(d)$ processes with non-integer values of d . As pointed out by Granger and Joyeux (1980), “*It was standard procedure to consider differencing time series to achieve stationarity*”—thus obtaining a form of the series that can be identified as an $ARMA$ model—however, “*Some econometricians were reluctant to this technique, believing that they may be losing information, by zapping out the low frequency components.*” But using infinite variance series without differencing them was also a source of difficulties at that time. Fractional integration encompasses $ARMA$ models for $d = 0$, and $ARIMA$ models for $d = 1$. The process is stationary and ergodic when $d \in (-\frac{1}{2}, \frac{1}{2})$; nonstationary but mean-reverting⁷ when $\frac{1}{2} < d < 1$, and nonstationary and mean averting when $d \geq 1$.

Mean stationary processes (DGPs 2 and 3) have been used to model the behavior of real exchange rates, unemployment rates, current account, and several great ratios, such as the output-capital ratio and the consumption-income ratio. Unemployment has also been conceived as a nonstationary fractionally integrated process (Arino and Marmol 2004). Some examples can be found in Perron and Vogelsang (1992), Wu (2000), Wu, Chen, and Lee (2001) and D’Adda and Scorcu (2003). Trend Stationarity and $I(2)$ processes (DGPs 4 and 9, respectively) have been used to model growing variables, real and nominal, such as output, consumption and prices; several macro variables have been conceived as DGPs 5, 6, and 7 (Perron 1989, Perron 1997, Lumsdaine and Papell 1997, Mehl 2000). Variables identified as $I(2)$ processes can also be found

in Juselius (1996), Juselius (1999), Haldrup (1998), Muscatelli and Spinelli (2000), Coenen and Vega (2001), and Nielsen (2002). In Table (11) of appendix B, a few more examples are provided, in which the link between time-series econometrics and important economic issues is acknowledged.

3 Spurious regression since the roaring twenties

We now begin our survey of the development of the theory of spurious regression. The related literature is vast, for which reason we focus mainly on a limited selection of articles which, in our view, are particularly representative.⁸ Unless otherwise specified, the regression specification for which all asymptotics are presented is hereinafter expression (1). We focus mainly on the rate of divergence of the relevant t-ratios and let aside in most cases the asymptotic distributions that would be obtained had the statistics being correctly normalized. This is so because such distributions do have nuisance parameters that prevent one from making use of these to do correct inference; moreover, practitioners cannot be aware *a priori* of the adequate normalization. However, in some cases (Kim, Lee, and Newbold 2004, Sun 2004, Moon, Rubia, and Valkanov 2006) the asymptotic distributions are important because of the relevance for practitioners.⁹

3.1 Yule's experiment

Spurious correlation was evidenced by Yule (1926) in a computerless Monte Carlo experiment. Shuffling decks of playing cards, Yule obtained independent series of random numbers. In fact, Yule generated independent $I(0)$, $I(1)$, and $I(2)$ series and computed correlation coefficients amongst them. Such correlation coefficients provided correct inference when using $I(0)$ series, but became nonsensical when the order of integration of the variables was higher. With independent $I(0)$ variables the correlation coefficient remained close to zero. The same estimate achieved using $I(1)$ independent

variables no longer worked; many times it was close to unity, resulting in what we now refer to as a spurious correlation. If variables were $I(2)$, the most probable outcome was actually a correlation coefficient of close to 1: the spurious phenomenon was even stronger.

3.2 Reappraisal in the seventies: spurious Least Squares

Throughout most of the past century, it was commonly recommended to first-difference the series if these seemed to have a trending mechanism. However, not every econometrician was in agreement with such a method because, it was argued, differencing causes losses in the information contained in the original series. A profound reappraisal of this issue began with Granger and Newbold's (1974) article, which allowed the spurious regression in Least squares estimators to be identified. The Monte Carlo experiment described above revealed, among other things, that high R^2 and low Durbin-Watson statistics (hereinafter DW) should be considered as a sign of misspecification. They also pointed out, comparing the outcome of simulation with many results in applied econometrics, that problems of misspecification seemed to be widespread. It was proposed that first-differenced series be used, although the authors warned about the risks of catch-all solutions; their results may be considered as the seed of many fruitful extensions in time series econometrics.

3.3 Theory at last: asymptotics in nonsensical regressions

Phillips (1986) proposed the theoretical framework necessary to allow an understanding of Granger and Newbold's earlier results and provided a first insight into the phenomenon of spurious regression. His development set the groundwork for the spurious regression literature in econometrics. Whilst Granger and Newbold used an *i.i.d.* noise in their simulations, Phillips allowed a flexible autocorrelation structure, as well as some degree of heterogeneity. He then proved that, when specification (1) is estimated

the following asymptotics are obtained:

$\hat{\beta} = O_p(1)$	$\hat{\alpha} = O_p\left(T^{\frac{1}{2}}\right)$	$R^2 = O_p(1)$
$t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$	$t_{\hat{\alpha}} = O_p\left(T^{\frac{1}{2}}\right)$	$\mathcal{DW} \xrightarrow{p} 0$

Table 2: Orders in Probability: variables y_t and x_t both independently generated by DGP 6

where $O_p(m)$ denotes the order of magnitude and \xrightarrow{p} means convergence in probability. The most relevant results are that the R^2 does not collapse, and, the t-statistic associated to $\hat{\beta}$ diverges at a rate $T^{\frac{1}{2}}$. This means that the t-statistic will exceed the classical 5% – level critical values (–1.96 and 1.96) as the sample size grows and, with a sufficiently large sample, the null hypothesis of the t-statistic ($\mathcal{H}_0 : \beta = 0$) will be rejected with certainty.

Park and Phillips (1989) demonstrated the presence of spurious regression when the independent variables are $I(2)$. Marmol (1995) proved that the phenomenon of spurious regression occurs in a more general nonstationary framework; he demonstrated that spurious regression occurs when independent series x_t, y_t are integrated of order d for $d \in \mathcal{N} = \{1, 2, \dots\}$ (see DGP 9). Empirically relevant DGPs, such as the $I(2)$ process (identified, as mentioned earlier, with uncontrolled inflation) can produce spurious regression. The asymptotic results are somewhat similar to those of Phillips:

$\hat{\beta} = O_p(1)$	$\hat{\alpha} = O_p\left(T^{\frac{1}{2}-d}\right)$	$R^2 = O_p(1)$
$t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$	$t_{\hat{\alpha}} = O_p\left(T^{\frac{1}{2}}\right)$	$\mathcal{DW} = O_p\left(T^{-2}\right)$

Table 3: Orders in Probability: variables y_t and x_t both independently generated by DGP 9. Both variables are integrated of the same order.

Although the parameter estimate of the constant term, $\hat{\alpha}$, diverges at a different rate, the t-statistic associated with $\hat{\beta}$ diverges at the same rate as in Phillips’s (1986) seminal

work, i.e. $O_p\left(T^{\frac{1}{2}}\right)$.

One limitation of Marmol's (1995) results is that both the dependent and the independent variables share the same order of integration. Banerjee, Dolado, Galbraith, and Hendry (1993), provided Monte Carlo evidence of spurious regression when the order of integration of each variable is different. Marmol (1996), later allowed the relevant DGPs for both y and x to be integrated of different orders; $y_t \sim I(d_1)$ and $x_t \sim I(d_2)$ where d_1 and $d_2 \in N = \{1, 2, \dots\}$. Marmol's (1996) results are twofold; either $d_1 > d_2$ or vice versa:

- When $d_1 > d_2$:

$\hat{\beta} = O_p\left(T^{d_1-d_2}\right)$	$\hat{\alpha} = O_p\left(T^{d_1-\frac{1}{2}}\right)$
$t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$	$t_{\hat{\alpha}} = O_p\left(T^{\frac{1}{2}}\right)$
$\mathcal{DW} = O_p\left(T^{-1}\right)$ if $d_2 = 1$	$\mathcal{DW} = O_p\left(T^{-2}\right)$ if $d_2 = 2, 3, \dots$

Table 4: Orders in Probability: variables y_t and x_t both independently generated by DGP 9. Variables are integrated of different orders. $d_1 > d_2$

- When $d_1 < d_2$:

$\hat{\beta} = O_p\left(T^{d_1-d_2}\right)$	$\hat{\alpha} = O_p\left(T^{d_1-\frac{1}{2}}\right)$
$t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$	$t_{\hat{\alpha}} = O_p\left(T^{\frac{1}{2}}\right)$
$\mathcal{DW} = O_p\left(T^{-1}\right)$ if $d_1 = 1$	$\mathcal{DW} = O_p\left(T^{-2}\right)$ if $d_1 = 2, 3, \dots$

Table 5: Orders in Probability: variables y_t and x_t both independently generated by DGP 9. Variables are integrated of different orders. $d_1 < d_2$

- In both cases $R^2 = O_p(1)$

Indeed, spurious regression persists in the presence of the so-called unbalanced regressions. The rate of divergence of $t_{\hat{\beta}}$ remains $T^{\frac{1}{2}}$. de Jong (2003) extended the study of spurious regression using independent driftless unit root processes; he used DGP (6) to generate the series and ran the following specification, which operates with logarithmic

transformation of both variables—a direction which has proved extremely relevant for empirical purposes:¹⁰

$$\ln(y_t) = \alpha + \beta \ln(x_t) + u_t$$

de Jong discovered similar results to previous works, i.e. $\hat{\beta} = O_p(1)$ and $t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$.

Entorf (1997) made a slight-and yet, fundamental-modification to the Phillips' DGPs by adding a drift (DGP 7).¹¹ Amongst the most relevant consequences of such drift is the fact that there is not only a stochastic trend but also a deterministic one. In the long run, the deterministic trend dominates the stochastic (see Appendix C). The asymptotic results of estimating equation (1) using independent variables generated by DGP 7 are:

$$\frac{t_{\hat{\beta}} = O_p(T)}{(1 - R^2) = O_p(T^{-1})} \quad \frac{t_{\hat{\alpha}} = O_p\left(T^{\frac{1}{2}}\right)}{\mathcal{DW} = O_p(T^{-1})}$$

Table 6: Orders in Probability: variables y_t and x_t both independently generated by DGP 7.

Note that $t_{\hat{\beta}}$ grows at rate T instead of $T^{\frac{1}{2}}$, contrary to the results presented so far, due to the presence of a deterministic trend.

3.4 Spurious regression and long memory: an unforgettable extension

Among the first papers to deal with spurious regression in econometrics¹² using long memory processes are those of Cappuccio and Lubian (1997) and Marmol (1998). The authors use the nonstationary fractionally-integrated processes specified in DGP 11. Under these conditions, the asymptotics of an LS regression as specified in expression (1) are as follows:¹³

$\hat{\beta} = O_p(1)$	$t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$
$R^2 = O_p(1)$	$\mathcal{DW} = O_p\left(T^{-(1+2d)}\right)$

Table 7: Orders in Probability: variables y_t and x_t both independently generated by DGP 11. The variables are integrated of the same order.

Note that as most of the previous cases the t-statistic of $\hat{\beta}$ diverges at rate $T^{\frac{1}{2}}$. The main difference lies in the \mathcal{DW} , the rate of divergence of which varies according to the degree of long memory, measured by d . These results can be understood as an argument against the usefulness of the ‘rule-of-thumb’; a regression was usually considered spurious when $R^2 > \mathcal{DW}$. However, when long memory is present in the variables, the regression may well be spurious even if $R^2 < \mathcal{DW}$. It is worth mentioning that the fractional integration parameter, d , is the same in both variables. This ‘shortcoming’ is fixed by Tsay and Chung (2000), who used a manifold approach. They used two DGPs (No 10 and 11) and then combined these in a simple regression. Thus, there are four fractionally-integrated processes, two stationary, $[FI(d_1)$ and $FI(d_2)$ with $d_i \in (0, \frac{1}{2})]$, and two nonstationary, $[FI(d_i + 1)$ for $d_i \in (\frac{1}{2}, \frac{3}{2})]$. The authors then studied several specifications using six DGPs. The first four are used in the estimation of specification (1) whilst the remaining two estimate $y_t = \alpha + \delta t + u_t$:

1. $y_t \sim FI(1 + d_1)$ and $x_t \sim FI(1 + d_2)$,¹⁴
2. $y_t \sim FI(d_1)$ and $x_t \sim FI(d_2)$ where $d_1 + d_2 > \frac{1}{2}$,
3. $y_t \sim FI(1 + d_1)$ and $x_t \sim FI(d_2)$ where $d_2 > \frac{1}{2}$,
4. $y_t \sim FI(d_1)$ and $x_t \sim FI(1 + d_2)$ where $d_1 > \frac{1}{2}$,
5. $y_t \sim FI(1 + d_1)$ where $d_1 > 0$,
6. $y_t \sim FI(d_1)$ where $d_1 > 0$.

We can summarize the results by stating that $t_{\hat{\beta}} = O_p(T^r)$ where $0 < r < 1$ which means that the t-ratio always diverges; the divergence rate, r , is, $T^{\frac{1}{2}}$, $T^{d_1+d_2-\frac{1}{2}}$, T^{d_2} , T^{d_1} , $T^{\frac{1}{2}}$, and T^{d_1} for cases 1 – 6.

Special attention should be given to the Durbin-Watson statistic, which collapses to zero (as usual) except in the second case, where $DW = O_p(1)$ and hence does not converge to zero as it does elsewhere. This also could be interpreted as an important indication of the need for caution when using the rule-of-thumb, which states that when $R^2 > DW$ there may be spurious regression. As in Cappuccio and Lubian (1997), there is evidence that the rule-of-thumb may be a dangerous tool. Another important result appears when we compare model 1 and model 2. The reader may notice that the variables used in the second model are merely the first differences of those used in the first. What is surprising is that the spurious regression persists after the variables have been differenced.¹⁵ This goes against fifty years of tradition (differencing is used to deal with spurious regression¹⁶). Tsay and Chung (2000) actually go further by suggesting that the spurious phenomenon is due to the long memory properties of the series and not to the presence of unit roots.¹⁷ Sun (2006) shows that spurious regression can occur between two stationary generalized fractional processes, as long as their generalized fractional differencing parameters sum up to a value greater than $\frac{1}{2}$ and their spectral densities have poles at the same location.

3.5 Spurious regression with stationary series: size-matters

Granger, Hyung, and Jeon (2001) show, both through Monte Carlo experiments and theoretically that the spurious regression phenomenon may occur amongst independent stationary series.¹⁸ Two independent $AR(1)$ series (see DGP 2 with $P_w = 1$) were run together to estimate regression (1), resulting in a rejection rate of the t-statistic greater than the expected 5%. Granger, Hyung, and Jeon (2001) provide a theoretical proof of why this happens (although the variance of the estimates does not diverge, it may not be unity, depending on the values of the DGP's parameters).¹⁹

These results are extended to long-span MA processes (see DGP 1). Curiously, the spurious regression does not depend on the sample size, but rather on the MA span

parameter, q_w . What is interesting about this result is the fact that it better explains the spurious regression phenomenon in small-sized samples. This would complete the theoretical framework necessary to allow an understanding of the Monte Carlo results of Granger and Newbold (1974) and Ferson, Sarkissian, and Simin (2003)

Granger, Hyung, and Jeon's (2001) results differ from all others, the effect being discussed is not an asymptotic phenomenon but rather a size distortion. Size distortions arise because standardizing a test statistic is difficult unless the exact form of the spectral density of the residuals is known. The intuition behind the spurious regression phenomenon is thus, different from the one that underlies all other results.

Mikosch and de Vries (2006) provide an alternative theory to explain spurious-type behavior akin to the financial risk measurement literature.²⁰ In the words of Mikosch and de Vries (2006):

“Estimators of the coefficients in equations of regression type which involve financial data are often found to vary considerably across different samples. This observation pertains to finance models like the CAPM beta regression, the forward premium equation and the yield curve regression. In economics, macro models like the monetary model of the foreign exchange rate often yield regression coefficients which significantly deviate from the unitary coefficient on money which is based on the theoretical assumption that money is neutral.”

Mikosch and de Vries (2006) prove that, when the distribution of the innovations is heavy tailed, that is, when there is a departure from the normality assumption usually made, using standard statistical tools (such as LS, for example) can be misleading.²¹

3.6 The last newcomer: Trend Stationarity

Hassler (2000) studied the spurious regression phenomenon from a different perspective. He considered the possibility of spurious regression when the variables do have a deterministic trend component (what we have defined earlier as trend stationarity [DGP

4]). Thus, there are two independent nonstationary variables. Hassler's (2000) results, as with those of Kim, Lee, and Newbold (2004), who worked with the same DGPs, are as follows:

$$\overline{t_{\hat{\beta}} = O_p\left(T^{\frac{3}{2}}\right)} \quad \overline{\beta = O_p(1)} \quad \overline{T^2(1 - R^2) \xrightarrow{p} 0}$$

Table 8: Orders in Probability: variables y_t and x_t both independently generated by DGP 4.

In addition, Kim, Lee, and Newbold (2004) proved that when one of the deterministic trend components is taken out (that is, either $\beta_y = 0$ and $\beta_x \neq 0$ or vice versa), the t-ratios converge to centered normal distribution, although its variance is not unity, which may provoke spurious rejection of the null hypothesis that the parameter is zero. That means that running a trend stationary variable (DGP 4) on a mean stationary variable (DGP 2) or vice versa lessens the "spuriousity" in the regression. The authors also generated independent series with the DGP (4), as did Entorf (1997), although they ran a different specification:

$$y_t = \alpha + \beta x_t + \delta t + u_t \quad (2)$$

The asymptotics provided in Kim, Lee, and Newbold (2003) only concern $\hat{\beta}$ and $t_{\hat{\beta}}$. We computed the orders in probability of the other two parameters, given their relevance. We included in Appendix D a guide on how to obtain these asymptotics:

$$\begin{array}{cc} \overline{\hat{\alpha} = O_p(1)} & \overline{t_{\hat{\alpha}} = O_p\left(T^{\frac{1}{2}}\right)} \\ \overline{\hat{\beta} = O_p\left(T^{-\frac{1}{2}}\right)} & \overline{t_{\hat{\beta}} = O_p(1)} \\ \overline{\hat{\delta} = O_p(1)} & \overline{t_{\hat{\delta}} = O_p\left(T^{\frac{1}{2}}\right)} \end{array}$$

Table 9: Orders in Probability: variables y_t and x_t both independently generated by DGP 4. The estimated specification is eq. (2).

It should be noted that the t-ratio converges to a normal distribution. However, since the variance is not unity, spurious regression may well be still present.

A relevant extension of Hassler (2000) and Kim, Lee, and Newbold (2004) was provided by Noriega and Ventosa-Santaulària (2006b) where (possibly multiple) structural breaks are added to the Trend Stationary DGP, i.e. DGP 5. Adding breaks to the specification of the DGPs makes the divergence rate of the t-statistic associated with $\hat{\beta}$ return to the usual $T^{\frac{1}{2}}$ ‘norm’. They also proved that adding a deterministic trend into the regression specification (see equation 2) does not prevent the phenomenon of spurious regression; $t_{\hat{\beta}}$ remains $O_p\left(T^{\frac{1}{2}}\right)$. In an effort to unify the related literature, Noriega and Ventosa-Santaulària (2007) filled a number of gaps until then unaddressed. They studied several previously overlooked combinations of DGPs and summarized most of the others’ results. DGPs 2-9 may indistinctly generate x and/or y .²² In fact, amongst the new possible combinations, the divergence rate of $t_{\hat{\beta}}$ usually remains $T^{\frac{1}{2}}$. The combination of a Trend Stationary and an $I(1)$ plus drift stands out since the divergence rate of the t-ratio is T rather than $T^{\frac{1}{2}}$ as in Kim, Lee, and Newbold (2004), although this result should have been anticipated given the asymptotic dominance of the deterministic trend over the stochastic. This combination is thus clearly linked to Entorf’s (1997) results.

Granger, Hyung, and Jeon (2001) showed that the use of an Heteroskedasticity and Autocovariance covariance matrix diminishes size distortions in some cases but their Monte Carlo evidence also showed that this is true only when the sample size is large (greater than 500). The use of HAC standard errors²³ when the DGPs are stationary or have a deterministic trend is not so obvious. We performed a simple simulation to support this view (see table 10). The variables x and y are independently generated either by DGP (4), (5), or (6). Lag selection was done as in Granger, Hyung, and Jeon (2001), that is, using the formula: $l = \text{integer} \left[4(T/100)^{1/4} \right]$. Simulation results show that size distortions are less severe when HAC is used, but remain extremely

high (perhaps results would improve further if the data could follow a pre-whitening procedure).

DGPs		LS standard errors				HAC standard errors			
x_i	y_i	Sample size							
		50	100	200	500	50	100	200	500
4	4	0.21	0.99	1.00	1.00	0.27	0.99	1.00	1.00
5	5	0.20	0.99	1.00	1.00	0.27	0.99	1.00	1.00
6	4	0.45	0.80	0.90	0.95	0.46	0.71	0.79	0.86
6	5	0.65	0.84	0.91	0.95	0.62	0.74	0.80	0.86
6	6	0.68	0.76	0.83	0.90	0.57	0.61	0.67	0.77

Table 10: t-ratio rejection rates using a 1.96 critical value (5% level) for a standard normal distribution: spurious regression using LS and HAC standard errors.

3.7 Next of kin: statistical tests, Long Horizon, and Instrumental Variables

Not all work on the spurious phenomenon has been monopolized by the LS simple estimator. Stochastic trending mechanisms have been used in most of the studies, although some exceptions are presented at the end of this section. Equally relevant are the Instrumental Variables (IV) estimates, which have also been analyzed (Ventosa-Santaulària 2009).

Giles (2007) studied two important residual tests: the Jarque-Bera test [hereinafter \mathcal{JB} ; see Jarque and Bera (1980)] and the Breusch-Pagan-Godfrey test (Breusch and Pagan 1980, Godfrey 1988), used to test for normality and autocorrelation/heteroskedasticity evidence in the residuals of an LS regression, respectively; he used independent driftless random walks (DGP 6) and proved that both statistics diverge at rate T , that is:

- Jarque-Bera test: $\mathcal{JB} = O_p(T)$
- Breusch-Pagan-Godfrey test: $(T \cdot R^2) = O_p(T)$

The null hypothesis of normality or serial independence/homoskedasticity will be eventually rejected even if it is correct, given a large enough sample.²⁴ Furthermore,

Ventosa-Santaulària and Vera-Valdés (2008) studied the behavior of the classical Granger-Causality test [see Granger (1969)], hereinafter GC. It is proved that the classical GC test fails to accept the null hypothesis of no GC between independent broken-trend (DGP 5) or broken-mean (DGP 3) processes whether the former series are differenced or not.

Kim, Lee, and Newbold (2005) study several important nonlinearity tests: the RESET test (Ramsey 1969), the McLeod and Li test (McLeod and Li 1983), the Keenan test (Keenan 1985), the Neural Network test (White 1989), White's Information test (White 1992), and the Hamilton test (Hamilton 2000). All these tests were studied by way of finite-sample experiments using independent driftless random walks (see DGP 6). They also presented the asymptotics of the RESET and Keenan tests; both test-statistics are $O_p(T)$, so the null of linearity is eventually rejected as the sample size grows. In the same vein, Noriega and Ventosa-Santaulària (2006a) proved that, when the variables are generated independently by any combination of DGPs (3), (5), (7), and (8), the Engle and Granger cointegration-test spuriously rejects the null of no cointegration²⁵ in an indeterminate number of cases since the relevant t-statistic is $O_p\left(T^{\frac{1}{2}}\right)$.²⁶

Spurious regression has also been identified in the context of Long-Horizon (henceforth LH) regressions, which are used in situations where previous “short-term”²⁷ studies have failed. In fact, Valkanov's (2003) insight is that rolling summations of $I(0)$ variables, that is LH variables, behave asymptotically as $I(1)$ series. Hence, the theory of spurious regression between independent $I(1)$ variables, discussed earlier, should give readers the intuition. Late eighties' studies provided interesting results in economics and finance. As asserted by Valkanov (2003):

“The results [...] are based on long-horizon variables, where a long-horizon variable is obtained as a rolling sum of the original series. It is heuristically argued that long-run regressions produce more accurate results by strengthening the signal coming from the data, while eliminating the noise”.

More precisely, if the specification to be estimated is equation (1), a Long-Horizon “reinterpretation” is carried out with the building of partially aggregated variables. As usual, let, $w = x, y$, then $w_t^k = \sum_{j=0}^{k-1} w_{t+j}$. The LH regression specification can usually be one of the following:

$$\begin{aligned} y_t^k &= \alpha + \beta x_t^k + u_t \\ y_t^k &= \alpha + \beta x_{t-k}^k + u_t \\ y_t^k &= \alpha + \beta x_t + u_t \end{aligned} \tag{3}$$

Such specifications are used mostly in the estimation of the equity/dividend relationship and to test both the neutrality of money or the Fisher effect. Based upon the previous specifications, Valkanov (2003) and Moon, Rubia, and Valkanov (2006) proved that this regression strategy also presents the spurious regression phenomenon. To do this, they let time overlap in the summations as a fixed fraction of the sample size, $k = [\lambda \cdot T]$ where $\lambda \in [0, 1]$. Valkanov (2003) then defines the following DGP:

$$y_t = \beta x_t + u_{yt} \tag{4}$$

where $(1 - L)\phi(L)x_t = u_{xt}$. The variable x_t follows an autoregressive process whose highest root is unity whilst the rest, represented in the polynomial $\phi(L)$, is invertible. Let $\omega = (u_{yt}, u_{xt})'$ be defined as a martingale difference sequence with $E(\omega_t \omega_t' | \omega_{t-1} \dots) = [\sigma_{11}^2 \ \sigma_{12}; \sigma_{21} \ \sigma_{22}^2]$ and finite fourth moment. We could argue that, when $\beta = 0$, the t-ratio associated with its estimate should be small enough for the null hypothesis to be accepted. As in previous results, that does not happen; $t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$. Valkanov (2003) and Moon, Rubia, and Valkanov (2006) suggest the use of a rescaled t-ratio, $\frac{t_{\hat{\beta}}}{T^{\frac{1}{2}}}$. Although the limiting distribution of such t-statistic's is neither normal nor pivotal, it can be easily simulated and hence used. Lee (2006) extends the results for fractionally integrated processes and finds that the t-ratio asso-

ciated with $\hat{\beta}$ in equation (3) diverges: $t_{\hat{\beta}} = O_p\left(T^{\frac{1}{2}}\right)$.

Ventosa-Santaulària (2009) studies the asymptotics of Instrumental Variables (IV) using DGPs (5) and (7), not only for x and y , but also for a spurious and independent instrument. It is shown that the t-ratio still diverges at rate $T^{\frac{1}{2}}$ which confirms the Monte Carlo simulations carried out by Leybourne and Newbold (2003). This result can be also seen as a complement to those presented by Phillips and Hansen (1990) and Hansen and Phillips (1990), in which the use of spurious instruments is proposed to improve the estimates whenever strong endogeneity is present between x and the residual term in cointegrated relationships.

Sun (2004) developed a convergent t-statistic for correcting the phenomenon of spurious regression. The new t-ratio is based on an estimate of the parameter variance made in the same manner as HAC standard errors. This t-statistic converges to a non-degenerate limiting distribution for many cases of spurious regression. He considered the regression between two independent nonstationary $I(d)$ processes with $d > \frac{1}{2}$ as well as the regression between an independent nonstationary $I(d)$ process and a linear trend [see equation (5)]:

$$y_t = \alpha + \delta t + u_t \quad (5)$$

In previous studies, the presence of unit roots in the series generally produced a $T^{\frac{1}{2}}$ divergent t-ratio. To avoid such divergence, Sun proposes a rescaling of the parameter estimate using a new standard error. The new Standard Error is computed in the same way HAC is. The main difference is that HAC estimates usually require a bandwidth or truncation lag. Sun suggests using the entire sample length:

$$\hat{\sigma}_{\hat{\beta}}^2 = \left(\sum_{t=1}^T (x_t - \bar{x})^2 \right)^{-1} \cdot T \hat{\Omega} \cdot \left(\sum_{t=1}^T (x_t - \bar{x})^2 \right)^{-1}$$

where

$$\hat{\Omega} = \sum_{j=-T+1}^{T-1} \kappa\left(\frac{j}{T}\right) \hat{\Gamma}(j)$$

$$\hat{\Gamma}(j) = \begin{cases} \frac{1}{T} \sum_{t=1}^{T-j} (x_{t+j} - \bar{x}) \hat{u}_{t+j} \hat{u}_t (x_t - \bar{x}) & \text{for } j \geq 0 \\ \frac{1}{T} \sum_{t=-j+1}^T (x_{t+j} - \bar{x}) \hat{u}_{t+j} \hat{u}_t (x_t - \bar{x}) & \text{for } j < 0 \end{cases}$$

where κ is a kernel function that belongs to a class that ensures positive definitiveness. Whether $x_t \sim I(d)$ or $x_t = t$, well-defined asymptotic distributions for the t-ratio are provided.

4 What to do if one fears spurious regression

The spurious regression phenomenon pervades many subfields in time series analysis. It might be controlled by using correctly scaled t-ratios, as suggested earlier, but having a clear idea of what DGP best emulates the properties of the series (this could be labeled as *DGPi-fication*) would therefore be necessary (examples of this can be: (i) evidence of UR must be obtained before a cointegration analysis is undertaken; (ii) the nature of the trending mechanism should be identified prior to the application of a transformation intended to render stationary the series, and; (iii) a test such as Robinson's (1994) could be undertaken in order to identify long-memory behavior). This can be achieved by means of applying a battery of tests to our series. Such an approach is not exempted of failures. Many statistical tests are known to yield spurious evidence under specific circumstances (see the previous subsection). Nevertheless, pretesting the series remains an adequate strategy and allows the practitioner to be aware of the potential difficulties he could face. In this section we include a short—and incomplete—list of tests that are employed in the *DGPi-fication* of the series:

1. Drawing inference concerning the nonstationarity of the series can be done by

means of Dickey-Fuller-type (DF) tests [see Dickey and Fuller (1979) and Dickey and Fuller (1981)]. The original DF test distinguishes between the null hypotheses of UR (DGPs 6 and 7) and the alternatives of stationarity (DGPs 1, 2 and 4). Other well-known UR tests are: (i) the KPSS test (Kwiatkowski, Phillips, Schmidt, and Shin 1992); (ii) the GLS-detrended DF test (Elliott, Rothenberg, and Stock 1996); (iii) the Phillips-Perron test (Phillips and Perron 1988), and; (iv) the Ng and Perron test (Ng and Perron 2001, Perron and Ng 1996).

2. The UR tests previously mentioned provide severely biased results under the hypothesis of trend-stationarity in the presence of structural breaks;²⁸ several alternatives are available. Perron (1989) suggested the use of a DF-type test with (point, level and trend) breaks specified in the auxiliary regression (see DGP 5); the break dates must be decided by the practitioner. Zivot and Andrews (1992) also proposed to modify the DF test in the same direction than Perron, only they allowed the break date to be endogenously specified; their test allows for a single break [see Lumsdaine and Papell (1997) for an extension that allows for two breaks] under the alternative hypothesis (DGP 5) and rules out the possibility of a break under the null hypothesis of UR (DGP 8); Carrion-i-Silvestre and Sansó (2006) proposed a test where a break under the null hypothesis is taken into account.
3. Bai and Perron (1998) proposed a test to distinguish between DGPs 2 and 4 and DGPs 3 and 5. The test presupposes that the trending mechanism is exclusively deterministic.
4. Long-range dependence: many testing procedures are also available. R/S-type test [see Hurst (1951), Mandelbrot and Taqqu (1979) and Lo (1991)] are commonly used to identify Long Memory (LM) against stationarity. However, Bhattacharya, Gupta, and Waymire (1983) proved that the classical R/S test may provide evidence of LM even if the series is stationary when the latter contains

a trending mechanism. Moreover, Mikosch and Stărică (2000) and Mikosch and Stărică (2004) proved that the sample autocorrelation function (sample ACF) can also be a misleading statistical tool when used to identify LM; stationary series that include a non-linear component might yield a sample ACF usually attributed to LM processes [see also Teverovsky and Taqqu (1997)]. Several Short memory processes may thus seem to behave as LM processes. This phenomenon can be labeled as spurious long memory (Mikosch and Stărică 2004).

5. Many other tests have been proposed to identify LM while they control for possible non-linearities (structural breaks in the mean or the variance of the series). See Liu, Pan, and Hsueh (1993), Robinson (1994), Lobato and Robinson (1996), Giraitis, Kokoszka, and Leipus (2001), Giraitis, Kokoszka, Leipus, and Teysiere (2003), Berkes, Horváth, Kokoszka, and Shao (2006), Zhang, Gabrys, and Kokoszka (2007), Aue, Horváth, Hušková, and Kokoszka (2008), and Jach and Kokoszka (2008).

Concluding remarks

Spurious regression can arise wherever a trending mechanism is present in the data. Even some stationary-autocorrelated processes cause spurious results.

Applied macroeconomists and financial experts have been steadily incorporating technical advances in the analysis of the spurious regression, a phenomenon identified for many empirically-relevant Data Generating Processes. These include stationary processes with AR (or long MA) structure and/or level breaks; random walks (with or without drifts), trend stationarity (with possible level and trend breaks), long-memory processes (whether stationary or not) and so on. These processes have been associated with unemployment rates, price levels, real exchange rates, monetary aggregates, gross domestic product and various financial variables. The use of Least Squares with such

variables entails a high risk of obtaining a spurious relationship.

Differencing the series may not always prevent spurious estimates; nor should the $R^2 > DW$ rule-of-thumb be seen as an adequate rule to identify a spurious regression. Cointegration analysis appears to better prevent non-sensical statistical relationships although, one should bear in mind Phillips's (2003) and further study the statistical relationship at hand. Out-of-sample forecasting evaluation could be an option. Most macroeconomic variables are either nonstationary or very persistent. Pre-testing the variables in order to identify the nature of the trending mechanism arises as the golden rule to avoid non-sense regression. Once the DGP is correctly identified, spurious regression is "easier" to deal with.

Attaining a clear understanding of any problem is the first step toward finding its solution.

References

- ADELMAN, I. (1965): "Long Cycles: Fact or Artifact," *American Economic Review*, 55, 444–463.
- AGHION, P., AND S. N. DURLAUF (2005): *Handbook of Economic Growth, Volume 1A*. North-Holland.
- ALDRICH, J. (1995): "Correlations genuine and spurious in Pearson and Yule," *Statistical Science*, 10(4), 364–376.
- ALOGOSKOUFIS, G., AND R. SMITH (1991): "The Phillips Curve, The Persistence of Inflation, and the Lucas Critique: Evidence from Exchange-Rate Regimes," *The American Economic Review*, 81(5), 1254–1275.
- ARINO, M., AND F. MARMOL (2004): "A permanent-transitory decomposition for ARFIMA processes," *Journal of Statistical Planning and Inference*, 124, 87–97.
- AUE, A., L. HORVÁTH, M. HUŠKOVÁ, AND P. KOKOSZKA (2008): "Testing for changes in polynomial regression," *Bernoulli*, 14(3), 637–660.
- BAI, J., AND P. PERRON (1998): "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica*, 66, 47–78.
- BANERJEE, A., J. DOLADO, J. GALBRAITH, AND D. HENDRY (1993): *Cointegration, Error Correction, and the Econometric Analysis of Non-Stationary Data*. Oxford University Press.

- BARDSEN, G., E. JANSEN, AND R. NYMOEN (2004): "Econometric Evaluation of the New Keynesian Phillips Curve," *Oxford Bulletin of Economics and Statistics*, 66(s 1), 671–686.
- BERKES, I., L. HORVÁTH, P. KOKOSZKA, AND Q. SHAO (2006): "On discriminating between long-range dependence and changes in mean," *Annals of Statistics*, 34(3), 1140–1165.
- BERNARD, A., AND S. DURLAUF (1995): "Convergence in International Output," *Journal of Applied Econometrics*, 10(2), 97–108.
- BERNARD, A., AND S. DURLAUF (1996): "Interpreting tests of the Convergence Hypothesis," *Journal of Econometrics*, 71, 161–173.
- BHATTACHARYA, R., V. GUPTA, AND E. WAYMIRE (1983): "The Hurst effect under trends," *Journal of Applied Probability*, pp. 649–662.
- BREIDT, F., N. CRATO, AND P. DE LIMA (1998): "The detection and estimation of long memory in stochastic volatility," *Journal of Econometrics*, 83(1-2), 325–348.
- BREUSCH, T., AND A. PAGAN (1980): "The Lagrange Multiplier Test and its Applications to Model Specification in Econometrics," *Review of Economic Studies*, 47, 239–254.
- CAMPELL, J., AND P. PERRON (1991): "What Macroeconomist Should Know about Unit Roots," in *NBER Macroeconomics Annual*, ed. by O. Blanchard, and S. Fisher, pp. 141–201. MIT Press.
- CAPPUCCIO, N., AND D. LUBIAN (1997): "Spurious Regressions Between I(1) Processes with Long Memory Errors," *Journal of Time Series Analysis*, 18, 341–354.
- CARRION-I-SILVESTRE, J., AND A. SANSÓ (2006): "Joint hypothesis specification for unit root tests with a structural break*," *Econometrics Journal*, 9(2), 196–224.
- CATI, R., M. GARCIA, AND P. PERRON (1999): "Unit roots in the presence of abrupt governmental interventions with an application to Brazilian data," *Journal of Applied Econometrics*, 14(1), 27–56.
- CAVALIERE, G., AND R. TAYLOR (2006): "Testing for a Change in Persistence in the Presence of a Volatility Shift," *Oxford Bulletin of Economics and Statistics*, 68(s1), 761–781.
- CHEUNG, Y. (1993): "Long memory in foreign-exchange rates," *Journal of Business & Economic Statistics*, pp. 93–101.
- COCHRANE, J. (1994): "Permanent and Transitory Components of GNP and Stock Prices," *The Quarterly Journal of Economics*, 109(1), 241–265.
- COE, P., AND J. NASON (2004): "Long-run monetary neutrality and long-horizon regressions," *Journal of Applied Econometrics*, 19(3), 355–373.

- COENEN, G., AND J. VEGA (2001): "The Demand for M3 in the Euro Area," *Journal of Applied Econometrics*, 16(6), 727–748.
- CORBAE, D., AND S. OULIARIS (1988): "Cointegration and Tests of Purchasing Power Parity," *The Review of Economics and Statistics*, 70(3), 508–511.
- CRATO, N., AND P. DE LIMA (1994): "Long-range dependence in the conditional variance of stock returns," *Economics Letters*, 45(3), 281–285.
- CROWDER, W. (1997): "The Long-Run Fisher Relation in Canada," *The Canadian Journal of Economics*, 30(4b), 1124–1142.
- CUDDINGTON, J., AND H. LIANG (2000): "Purchasing power parity over two centuries," *Journal of International Money and Finance*, 19(5), 753–757.
- CULVER, S., AND D. PAPELL (1997): "Is there a unit root in the inflation rate? Evidence from sequential break and panel data models," *Journal of Applied Econometrics*, 12(4), 435–444.
- D'ADDA, C., AND A. SCORCU (2003): "On the time stability of the output-capital ratio," *Economic Modelling*, 20(6), 1175–1189.
- DAVIDSON, J., D. HENDRY, F. SRBA, AND S. YEO (1978): "Econometric Modelling of The Aggregate Time Series Relationship between the consumers: Expenditure and Income in the United Kindom," *Economic Journal*, 88, 661–92.
- DE JONG, R. (2003): "Logarithmic Spurious Regressions," *Economics Letters*, 81(1), 13–21.
- DELOACH, S. (2001): "More Evidence in Favor of the Balassa-Samuelson Hypothesis," *Review of International Economics*, 9(2), 336–342.
- DIBA, B., AND H. GROSSMAN (1988): "Explosive Rational Bubbles in Stock Prices?," *The American Economic Review*, 78(3), 520–530.
- DICKEY, D., AND W. FULLER (1979): "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 74, 427–31.
- (1981): "Likelihood ratio statistics for autoregressive time series with a unit root," *Econometrica*, pp. 1057–1072.
- DICKEY, D., AND S. PANTULA (1987): "Determining the Order of Differencing in Autoregressive Processes," *Journal of Business & Economic Statistics*, 5(4), 455–461.
- DRINE, I., AND C. RAULT (2003): "Do panel data permit the rescue of the Balassa-Samuelson hypothesis for Latin American countries?," *Applied Economics*, 35(3), 351–359.

- DURLAUF, S. (1996): "On The Convergence and Divergence of Growth Rates," *The Economic Journal*, 106(437), 1016–1018.
- (2001): "Manifesto for a Growth Econometrics," *Journal of Econometrics*, 100(1), 65–69.
- DURLAUF, S., AND P. PHILLIPS (1988): "Trends versus Random Walks in Time Series Analysis," *Econometrica*, 56(6), 1333–1354.
- DURLAUF, S., AND D. QUAH (1998): *The New Empirics of Economic Growth*. National Bureau of Economic Research Cambridge, Mass., USA.
- ELLIOTT, G., T. ROTHENBERG, AND J. STOCK (1996): "Efficient tests for an autoregressive unit root," *Econometrica: Journal of the Econometric Society*, pp. 813–836.
- ENDERS, W., AND S. HURN (1994): "Theory and Tests of Generalized Purchasing-Power Parity: Common Trends and Real Exchange Rates in the Pacific Rim," *Review of International Economics*, 2(2), 179–190.
- ENGLE, R., AND C. GRANGER (1987): "Co-integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, 55, 251–276.
- ENTORF, H. (1997): "Random Walks Wiht Drifts: Nonsense Regression and Spurious Fixed-Effect Estimation," *Journal of Econometrics*, 80, 287–296.
- EVANS, M., AND K. LEWIS (1995): "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?," *The Journal of Finance*, 50(1), 225–253.
- FAMA, E. (1965): "The behavior of stock-market prices," *Journal of business*, 38(1), 34.
- FARIA, J., AND M. LEÓN-LEDESMA (2003): "Testing the Balassa-Samuelson Effect: Implications for Growth and the PPP," *Journal of Macroeconomics*, 25(2), 241–253.
- FAUST, J., AND E. LEEPER (1997): "When Do Long-Run Identifying Restrictions Give Reliable Results?," *Journal of Business & Economic Statistics*, 15(3), 345–353.
- FERSON, W., S. SARKISSIAN, AND T. SIMIN (2003): "Is Stock Return Predictability Spurious?," *Journal of Investment Management*, 1(3), 1–10.
- FUHRER, J., AND G. MOORE (1995): "Inflation Persistence," *The Quarterly Journal of Economics*, 110(1), 127–159.
- FURMAN, E., AND R. ZITIKIS (2008): "Weighted risk capital allocations," *Insurance Mathematics and Economics*, 43(2), 263–269.
- (2009a): "General Stein-Type Decompositions of Covariances: Revisiting the Capital Asset Pricing Model," Proceedings of the 58th Annual Meeting of the Midwest Finance Association, Vol.6. 8 pages. Chicago, Illinois, USA.

- (2009b): “Weighted Pricing Functionals,” Actuarial Research Clearing House (ARCH). pp. 31. Proceedings of the 43rd Actuarial Research Conference, University of Regina, Saskatchewan, Canada.
- GALÍ, J., M. GERTLER, AND J. LÓPEZ-SALIDO (2005): *Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve*. Banco de España.
- GARCIA, R., AND P. PERRON (1996): “An Analysis of the Real Interest Rate Under Regime Shifts,” *The Review of Economics and Statistics*, 78(1), 111–125.
- GILES, D. E. (2007): “Spurious Regressions with Time-Series Data: Further Asymptotic Results,” *Communications in Statistics-Theory and Methods*, 36(5), 967–979.
- GIRAITIS, L., P. KOKOSZKA, AND R. LEIPUS (2001): “Testing for long memory in the presence of a general trend,” *Journal of Applied Probability*, pp. 1033–1054.
- GIRAITIS, L., P. KOKOSZKA, R. LEIPUS, AND G. TEYSSIERE (2003): “Rescaled variance and related tests for long memory in volatility and levels,” *Journal of Econometrics*, 112(2), 265–294.
- GODFREY, L. (1988): *Misspecification Tests in Econometrics: The Lagrange Multiplier Principle and Other Methods*. Cambridge University Press.
- GRANGER, C. (1969): “Investigating Causal Relations by Econometric Models and Cross-spectral Methods,” *Econometrica*, 37(3), 424–438.
- (1980): “Long Memory Relationships and the Aggregation of Dynamic Models,” *Journal of Econometrics*, 14, 227–38.
- (1981): “Some Properties of Time Series Data and their Use in Econometric Model Specification,” *Journal of Econometrics*, 16, 121–130.
- GRANGER, C., N. HYUNG, AND Y. JEON (2001): “Spurious regressions with stationary series,” *Applied Economics*, 33(7), 899–904.
- GRANGER, C., AND R. JOYEUX (1980): “An Introduction to Long-Memory Time Series Models and Fractional Differencing,” *Journal of Time Series Analysis*, 1, 15–29.
- GRANGER, C., AND P. NEWBOLD (1974): “Spurious Regressions in Econometrics,” *Journal of Econometrics*, 2, 11–20.
- GRANGER, C., AND A. WEISS (1983): *Time Series Analysis of Error-Correction Models*. Academic Press.
- HALDRUP, N. (1998): “An Econometric Analysis of I (2) Variables,” *Journal of Economic Surveys*, 12(5), 595–650.
- HAMILTON, J. (1994): *Time Series Analysis*. Princeton.
- (2000): “A Parametric Approach to Flexible Nonlinear Inference,” *Econometrica*, 69, 801–812.

- HAMORI, S., AND A. TOKIHISA (1997): "Testing for a unit root in the presence of a variance shift," *Economics Letters*, 57(3), 245–253.
- HANSEN, B., AND P. PHILLIPS (1990): "Estimation and Inference in Models of Cointegration: A Simulation Study," *Advances in Econometrics*, 8, 225–248.
- HASSLER, U. (1996): "Spurious regressions when stationary regressors are included," *Economics Letters*, 50(1), 25–31.
- HASSLER, U. (2000): "Simple Regressions with Linear Time Trends," *Journal of Time Series Analysis*, 21, 27–32.
- HASSLER, U. (2003): "Nonsense Regressions Due to Neglected Time Varying Means," *Statistical Papers*, 44, 169–182.
- HENDRY, D. (1980): "Econometrics-Alchemy or Science?," *Economica*, 47(188), 387–406.
- HENDRY, D., AND G. ANDERSON (1977): "Testing Dynamic Specification in Small Simultaneous Systems: An Application to a Model of Building Society Behaviour in the United Kingdom," *Frontiers in Quantitative Economics*, 3, 361–383.
- HENDRY, D., AND G. MIZON (1978): "Serial Correlation as Convenience Simplification, Not a Nuisance: A comment on a Study of the Demand for Money by the Bank of England," *Economic Journal*, 88, 549–63.
- HENDRY, D., AND M. MORGAN (1995): *The Foundations of Econometric Analysis*. Cambridge University Press.
- HENDRY, D., A. PAGAN, AND J. SARGAN (1984): "Dynamic Specification," *Handbook of Econometrics*, 2, 1023–1100.
- HIPEL, AND MCLEOD (1978): "Preservation of the Rescaled Adjusted Range Parts 1, 2 and 3," *Water Resources Research*, 14, 491–518.
- HOSKING, J. (1981): "Fractional Differencing," *Biometrika*, 68, 165–76.
- HURST, H. (1951): "Long Term Storage Capacity Reservoirs," *Transactions of The American Society of Civil Engineers*, 116, 770–799.
- JACH, A., AND P. KOKOSZKA (2008): "Wavelet-domain test for long-range dependence in the presence of a trend," *Statistics*, 42(2), 101–113.
- JARQUE, C., AND A. BERA (1980): "Efficient Tests for Normality, Homoskedasticity and Serial Independence of Regression Residuals," *Economics Letters*, 6, 255–259.
- JEVONS, W. (1884): "Commercial Crises and Sunspots," *Investigations in Currencies and Finance (MacMillan, London)*, pp. 221–243.
- JUSELIUS, K. (1996): "A Structured VAR under Changing Monetary Policy," *Journal of Business and Economics Statistics*, 16, 400–412.

- (1999): “Price Convergence in the Medium and Long Run: An I (2) Analysis of Six Price Indices,” *Cointegration, Causality and Forecasting: A Festschrift to Clive Granger*, Oxford University Press, Oxford.
- KAO, C. (1999): “Spurious regression and residual-based tests for cointegration in panel data,” *Journal of Econometrics*, 90(1), 1–44.
- KEENAN, K. (1985): “A Tukey nonadditivity type test for time series nonlinearity,” *Biometrika*, 72, 39–44.
- KIM, T., S. LEE, AND P. NEWBOLD (2003): “Spurious Regressions With Processes Around Linear Trends or Drifts,” Discussion Papers in Economics.
- KIM, T., S. LEE, AND P. NEWBOLD (2004): “Spurious Regressions With Stationary Processes Around Linear Trends,” *Economics Letters*, 83, 257–262.
- KIM, T., S. LEE, AND P. NEWBOLD (2005): “Spurious Nonlinear Regressions in Econometrics,” *Economics Letters*, 87, 301–306.
- KIM, T., S. LEYBOURNE, AND P. NEWBOLD (2002): “Unit root tests with a break in innovation variance,” *Journal of Econometrics*, 109(2), 365–387.
- (2004): “Behaviour of Dickey-Fuller Unit-Root Tests Under Trend Misspecification,” *Journal of Time Series Analysis*, 25(5), 755–764.
- KING, R., C. PLOSSER, J. STOCK, AND M. WATSON (1991): “Stochastic Trends and Economic Fluctuations,” *The American Economic Review*, 81(4), 819–840.
- KOUSTAS, Z. (1996): “Unemployment hysteresis in Canada: an approach based on long-memory time series models,” *Applied Economics*, 28(7), 823–831.
- KWIATKOWSKI, D., P. PHILLIPS, P. SCHMIDT, AND Y. SHIN (1992): “Testing the null hypothesis of stationarity against the alternative of a unit root,” *Journal of econometrics*, 54(1-3), 159–178.
- LEE, J. (2006): “Fractionally integrated long horizon regressions,” *Studies in Nonlinear Dynamics & Econometrics*.
- LEGENDRE, A. (1805): *Nouvelles méthodes pour la détermination des orbites des comètes*. F. Didot.
- LEYBOURNE, S., T. C. MILLS, AND P. NEWBOLD (1998): “Spurious rejections by Dickey–Fuller tests in the presence of a break under the null,” *Journal of Econometrics*, 87(1), 191–203.
- LEYBOURNE, S., AND P. NEWBOLD (2000): “Behavior of Dickey-Fuller t-tests when there is a break under the alternative hypothesis,” *Econometric Theory*, 16(05), 779–789.
- LEYBOURNE, S., AND P. NEWBOLD (2003): “Spurious Rejections by Cointegration Tests Induced by Structural Breaks,” *Applied Economics*, 35, 1117–1121.

- LI, Q., AND D. PAPELL (1999): "Convergence of International Output Time Series Evidence for 16 OECD Countries," *International Review of Economics and Finance*, 8, 267–280.
- LIU, Y., M. PAN, AND L. HSUEH (1993): "A modified R/S analysis of long-term dependence in currency futures prices," *Journal of International Financial Markets, Institutions & Money*, 3(2), 97–113.
- LO, A. (1991): "Long-term memory in stock market prices," *Econometrica*, pp. 1279–1313.
- LOBATO, I., AND P. ROBINSON (1996): "Averaged periodogram estimation of long memory," *Journal of Econometrics*, 73(1), 303–324.
- LOBATO, I., AND N. SAVIN (1998): "Real and spurious long-memory properties of stock-market data," *Journal of Business & Economic Statistics*, pp. 261–268.
- LUMSDAINE, R., AND D. PAPELL (1997): "Multiple Trend Breaks and the Unit-Root Hypothesis," *The Review of Economics and Statistics*, 79(2), 212–218.
- MANDELBROT, B., AND M. TAQQU (1979): "Robust R/S analysis of long run serial correlation," in *Proc. 42nd Session ISI*, pp. 69–99.
- MARMOL, F. (1995): "Spurious Regressions Between I(d) Processes," *Journal of Time Series Analysis*, 16, 313–321.
- MARMOL, F. (1996): "Nonsense Regressions between Integrated Processes of Different Orders," *Oxford Bulletin of Economics & Statistics*, 58(3), 525–36.
- MARMOL, F. (1998): "Spurious Regression Theory with Nonstationary Fractionally Integrated Processes," *Journal of Econometrics*, 84, 233–250.
- MCLEOD, A., AND W. LI (1983): "Diagnosing checking ARMA time series models using squared residual autocorrelation," *Journal of Time Series Analysis*, 4, 169–176.
- MEHL, A. (2000): "Unit Root Tests with Double Trend Breaks and the 1990s Recession in Japan," *Japan and the World Economy*, 12(4), 363–379.
- MIKOSCH, T. (2003): *Modeling dependence and tails of financial time series*. Chapman & Hall/CRC, Extreme Values in Finance, Telecommunications, and the Environment.
- MIKOSCH, T., AND C. DE VRIES (2006): "Tail Probabilities for Regression Estimators," Laboratory of Actuarial Mathematics, University of Copenhagen.
- MIKOSCH, T., AND C. STĂRICĂ (2000): "Is it really long memory we see in financial returns," *Extremes and integrated risk management*, 12, 149–168.
- (2004): "Nonstationarities in financial time series, the long-range dependence, and the IGARCH effects," *Review of Economics and Statistics*, 86(1), 378–390.

- MONTAÑÉS, A., AND M. REYES (1998): "Effect of a Shift in the Trend Function on Dickey-Fuller Unit Root Tests," *Econometric Theory*, 14, 355–363.
- MONTAÑÉS, A., AND M. REYES (1999): "The asymptotic behaviour of the Dickey-Fuller tests under the crash hypothesis," *Statistics and Probability Letters*, 42(1), 81–89.
- MOON, R., A. RUBIA, AND R. VALKANOV (2006): "Long-Horizon Regressions when the Predictor is Slowly Varying," *Journal of Business and Economic Statistics*.
- MORGAN, M. (1990): *The History of Econometric Ideas*. Cambridge University Press.
- MUSCATELLI, V., AND F. SPINELLI (2000): "The long-run stability of the demand for money: Italy 1861–1996," *Journal of Monetary Economics*, 45(3), 717–39.
- NELSON, C., AND H. KANG (1981): "Spurious Periodicity in Inappropriately Detrended Time Series," *Econometrica*, 49(3), 741–751.
- NELSON, C., AND C. PLOSSER (1982): "Trends and Random Walks in Macroeconomic Time Series: some evidence and Implications," *Journal of Monetary Economics*, 10, 139–169.
- NEWBY, W., AND K. WEST (1987): "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix," *Econometrica: Journal of the Econometric Society*, pp. 703–708.
- NG, S., AND P. PERRON (2001): "Lag length selection and the construction of unit root tests with good size and power," *Econometrica*, pp. 1519–1554.
- NIELSEN, H. (2002): "An I (2) Cointegration Analysis of Price and Quantity Formation in Danish Manufactured Exports," *Oxford Bulletin of Economics and Statistics*, 64(5), 449–472.
- NORIEGA, A., AND D. VENTOSA-SANTAULÀRIA (2006a): "Spurious Cointegration: the Engle-Granger test in the presence of Structural Breaks," Banco de México Working Paper No 2006-12.
- (2006b): "Spurious Regression Under Broken Trend Stationarity," *Journal of Time Series Analysis*, 27, 671–684.
- (2007): "Spurious Regression And Trending Variables," *Oxford Bulletin of Economics and Statistics*, 7, 4–7.
- PARK, J., AND P. PHILLIPS (1989): "Statistical Inference in Regressions with Integrated Processes: Part 2," *Econometric Theory*, 5(1), 95–131.
- PEARSON, K. (1897): "On a Form of Spurious Correlation Which May Arise When Indices Are Used in the Measurement of Organs," *Proceedings of the Royal Society of London*, 60, 489–498.

- PEDRONI, P. (2004): "Panel Cointegration; Asymptotic and Finite Sample Properties of Pooled Time Series Tests with an Application to the Purchasing Power Parity Hypothesis," *Econometric Theory*, 20(3), 597–625.
- PERRON, P. (1989): "The Great Crash, The Oil Price Shock and the Unit Root Hypothesis," *Econometrica*, 57, 1361–1401.
- (1990): "Testing for a unit root in a time series with a changing mean," *Journal of Business & Economic Statistics*, pp. 153–162.
- (1997): "Further evidence on breaking trend functions in macroeconomic variables," *Journal of Econometrics*, 80(2), 355–385.
- PERRON, P., AND S. NG (1996): "Useful modifications to some unit root tests with dependent errors and their local asymptotic properties," *The Review of Economic Studies*, pp. 435–463.
- PERRON, P., AND T. VOGELSANG (1992): "Nonstationarity and Level Shifts with an Application to Purchasing Power Parity," *Journal of Business and Economic Statistics*, 10(3), 301–320.
- PHILLIPS, P. (1986): "Understanding Spurious Regressions in Econometrics," *Journal of Econometrics*, 33, 311–340.
- (1998): "New Tools for Understanding Spurious Regressions," *Econometrica*, 66, 1299–1325.
- (2001): "Trending Time Series and Macroeconomic Activity: some present and future challenges," *Journal of Econometrics*, 100, 21–27.
- (2003): "Laws and Limits of Econometrics*," *The Economic Journal*, 113(486), 26–52.
- (2005): "Econometric Analysis of Fisher's Equation," *The American Journal of Economics and Sociology*, 64(1), 125–68.
- PHILLIPS, P., AND B. HANSEN (1990): "Statistical Inference in Instrumental Variables Regression with I (1) Processes," *The Review of Economic Studies*, 57(1), 99–125.
- PHILLIPS, P., AND H. MOON (1999): "Linear Regression Limit Theory for Nonstationary Panel Data," *Econometrica*, 67(5), 1057–1111.
- PHILLIPS, P., AND P. PERRON (1988): "Testing for a Unit Root in Time Series Regression," *Biometrika*, 75, 335–346.
- PLOSSER, C., AND G. SCHWERT (1978): "Money, Income and Sunspots: measuring economic relationships and the effects of differencing," *Journal of Monetary Economics*, 4, 637–660.
- RAMSEY, J. (1969): "Tests for specification errors in classical linear least squares regression analysis," *Journal of the Royal Statistical Society B*, 31, 155–182.

- ROBINSON, P. (1994): "Efficient Tests of Nonstationary Hypotheses.," *Journal of the American Statistical Association*, 89(428).
- SARGAN, G. (1964): *Wages and prices in United Kingdom: a study in econometric methodology*. Butterworth, London.
- SEN, A. (2001): "Behaviour of Dickey–Fuller F-tests under the trend-break stationary alternative," *Statistics and Probability Letters*, 55(3), 257–268.
- (2003): "On unit-root tests when the alternative is a trend-break stationary process," *Journal of Business and Economic Statistics*, 21(1), 174–184.
- (2008): "Behaviour of Dickey–Fuller tests when there is a break under the unit root null hypothesis," *Statistics and Probability Letters*, 78(6), 622–628.
- STOCK, J. (1987): "Asymptotic properties of least squares estimators of cointegrating vectors," *Econometrica*, pp. 1035–1056.
- SUN, Y. (2004): "A Convergent t-statistic in spurious regressions," *Econometric Theory*, 20(05), 943–962.
- (2006): "Spurious regressions between stationary generalized long memory processes," *Economics Letters*, 90(3), 446–454.
- SUN, Y., AND P. PHILLIPS (2004): "Understanding the Fisher equation," *Journal of Applied Econometrics*, 19(7), 869–886.
- TAYLOR, J. (1979): "Estimation and Control of a Macroeconomic Model with Rational Expectations," *Econometrica*, 47(5), 1267–1286.
- TAYLOR, R. (2005): "Fluctuation Tests for a Change in Persistence," *Oxford Bulletin of Economics and Statistics*, 67(2), 207–230.
- TEVEROVSKY, V., AND M. TAQQU (1997): "Testing for long-range dependence in the presence of shifting means or a slowly declining trend, using a variance-type estimator," *Journal of Time Series Analysis*, 18, 279–304.
- TSAY, W., AND C. CHUNG (2000): "The spurious regression of fractionally integrated processes," *Journal of Econometrics*, 96(1), 155–182.
- VALKANOV, R. (2003): "Long-horizon regressions: theoretical results and applications," *Journal of Financial Economics*, 68(2), 201–232.
- VENTOSA-SANTAULÀRIA, D. (2009): "Spurious Instrumental Variables," Forthcoming: Communications in Statistics. Theory and Methods.
- VENTOSA-SANTAULÀRIA, D., AND J. VERA-VALDÉS (2008): "Granger-Causality in the presence of structural breaks," *Economics Bulletin*, 3(61), 1–14.
- WHITE, H. (1989): "An additional hidden unit test for neglected nonlinearity in multilayer feedforward networks," *Proceedings of the international joint conference on neural networks, Washington, DC (IEEE Press, New York)*, 2, 451–455.

- (1992): *Estimation, Inference and Specification Analysis*. New York. Cambridge University Press.
- WU, J. (2000): “Mean Reversion of the Current Account: Evidence from the Panel Data Unit Root Test,” *Economics Letters*, 66(2), 215–222.
- WU, J., S. CHEN, AND H. LEE (2001): “Are current account deficits sustainable? Evidence from panel cointegration,” *Economics Letters*, 72(2), 219–224.
- YULE, G. (1897): “On the theory of correlation,” *Journal of the Royal Statistical Society*, 60(4), 812–854.
- (1926): “Why do we Sometimes get Nonsense-Correlations between Time-Series?—A Study in Sampling and the Nature of Time-Series,” *Journal of the Royal Statistical Society*, 89(1), 1–63.
- ZHANG, A., R. GABRYS, AND P. KOKOSZKA (2007): “Discriminating between long memory and volatility shifts,” *Statistics*, 36(4), 253–275.
- ZHU, Z. (1998): “The random walk of stock prices: evidence from a panel of G-7 countries,” *Applied Economics Letters*, 5(7), 411–413.
- ZIVOT, E., AND D. ANDREWS (1992): “Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit Root Hypothesis,” *Journal of Business And Economic Statistics*, 10, 251–270.

A Spurious regression using independent UR processes

The theoretical explanation of the spurious regression phenomenon documented in Granger and Newbold (1974) was provided by Phillips (1986). We present here a simple version of Phillips's (1986) results. Assume two independent non-stationary unit root processes:

$$\begin{aligned}x_t &= x_{t-1} + u_{xt} \\ y_t &= y_{t-1} + u_{yt}\end{aligned}$$

where, for simplicity, $u_{zt} \sim \mathcal{N}(0, \sigma_z^2)$; $u_{xt} \perp u_{yt}$. By solving recursively both equations, we obtain:

$$\begin{aligned}x_t &= X_0 + \underbrace{\sum_{i=1}^{t-1} u_{xt-i}}_{\xi_{xt-1}} \\ y_t &= Y_0 + \underbrace{\sum_{i=1}^{t-1} u_{yt-i}}_{\xi_{yt-1}}\end{aligned}$$

where X_0 and Y_0 are initial conditions. It can be proved (see, for example, Hamilton (1994) p. 486 and p.548) that:

$$\begin{aligned}T^{-\frac{3}{2}} \sum_{t=1}^T \xi_{zt-1} &\xrightarrow{d} \sigma_z \int_0^1 \omega_z(r) dr \\ T^{-2} \sum_{t=1}^T \xi_{zt-1}^2 &\xrightarrow{d} \sigma_z^2 \int_0^1 [\omega_z(r)]^2 dr \\ T^{-2} \sum_{t=1}^T \xi_{xt-1} \xi_{yt-1} &\xrightarrow{d} \sigma_x \sigma_y \int_0^1 \omega_x(r) \omega_y(r) dr\end{aligned}$$

where $\omega_z(1)$ is a standard brownian motion and \xrightarrow{d} denotes convergence in law. Therefore (assume that $u_{z0} = 0$):

$$\begin{aligned}\sum z_t &= Z_0 T + \underbrace{\sum_{t=1}^T \xi_{zt-1}}_{O_p\left(T^{\frac{3}{2}}\right)} \\ \sum z_t^2 &= Z_0^2 T + \underbrace{\sum_{t=1}^T \xi_{zt-1}^2}_{O_p(T^2)} + \sum_{t=1}^T \xi_{zt-1} \\ \sum x_t y_t &= X_0 Y_0 T + X_0 \sum_{t=1}^T \xi_{yt-1} + Y_0 \sum_{t=1}^T \xi_{xt-1} + \underbrace{\sum_{t=1}^T \xi_{xt-1} \xi_{yt-1}}_{O_p(T^2)}\end{aligned}$$

The LS formula of $\hat{\beta}$ is:

$$\hat{\beta} = \frac{T \sum x_t y_t - \sum x_t \sum y_t}{T \sum x_t^2 - (\sum x_t)^2}$$

Replacing the sums that appear in this formula with the above asymptotic expressions and letting $T \rightarrow \infty$ yields:

$$\begin{aligned}\hat{\beta} &= \frac{\sigma_y \int_0^1 \omega_x(r) \omega_y(r) dr - \int_0^1 \omega_x(r) dr \int_0^1 \omega_y(r) dr}{\sigma_x \int_0^1 [\omega_x(r)]^2 dr - \left(\int_0^1 \omega_x(r) dr \right)^2} \\ &= O_p(1)\end{aligned}$$

The same can be done with the variance of the regression as well as with the t-ratio associated to $\hat{\beta}$:

$$\begin{aligned}\hat{\sigma}^2 &\xrightarrow{d} T^{-1} \left[\sum y_t^2 + \hat{\alpha}^2 T + \hat{\beta}^2 \sum x_t^2 - 2\hat{\alpha} \sum y_t - 2\hat{\beta} \sum x_t y_t + 2\hat{\alpha}\hat{\beta} \sum x_t \right] \\ &= O_p(T) \\ t_{\hat{\beta}} &= O_p(T^{\frac{1}{2}})\end{aligned}$$

Note that the t-ratio diverges as the sample size grows. The usual critical values to test the null hypothesis $\beta = 0$ are ± 1.96 (level: 5%) so it is straight forward to see that the null hypothesis will always be rejected for a sample sufficiently large: this is a simple example of spurious regression.

B Empirical work and DGP's

Subject or field	Selection of articles
GDP and GNP trending, growth, convergence and Business cycles, Output-Capital Ratio, exports, unemployment,	(Nelson and Plosser 1982, Perron 1989, King, Plosser, Stock, and Watson 1991, Zivot and Andrews 1992, Cochrane 1994, Bernard and Durlauf 1995, Bernard and Durlauf 1996, Durlauf 1996, Koustas 1996, Perron 1997, Durlauf and Quah 1998, Li and Papell 1999, Mehl 2000, Durlauf 2001, Nielsen 2002, D'Adda and Scorcu 2003, Arino and Marmol 2004, Aghion and Durlauf 2005)
Quantitative theory and Money neutrality, Demand of Money,	(Plosser and Schwert 1978, Campbell and Perron 1991, Faust and Leeper 1997, Muscatelli and Spinelli 2000, Coenen and Vega 2001, Valkanov 2003, Coe and Nason 2004, Moon, Rubia, and Valkanov 2006)
Purchasing Power Parity, Exchange Rates, Real interest Rate, Taylor Rule, Inflation, Persistence, Stock Prices, CAPM, Forward premium equation, Yield curve, Financial risk, Premium calculation principles (pcp's),	(Taylor 1979, Corbae and Ouliaris 1988, Diba and Grossman 1988, Perron and Vogelsang 1992, Cheung 1993, Crato and de Lima 1994, Enders and Hurn 1994, Juselius 1996, Garcia and Perron 1996, Cullver and Papell 1997, Breidt, Crato, and De Lima 1998, Lobato and Savin 1998, Zhu 1998, Cati, Garcia, and Perron 1999, Juselius 1999, Cuddington and Liang 2000, Mikosch 2003, Pedroni 2004, Taylor 2005, Cavaliere and Taylor 2006, Furman and Zitikis 2008, Furman and Zitikis 2009a, Furman and Zitikis 2009b)
Phillip's Curve, Fisher equation, Balassa-Samuelson Effect,	(Alogoskoufis and Smith 1991, Evans and Lewis 1995, Fuhrer and Moore 1995, Crowder 1997, DeLoach 2001, Drine and Rault 2003, Faria and León-Ledesma 2003, Bardsen, Jansen, and Nymoen 2004, Sun and Phillips 2004, Galí, Gertler, and López-Salido 2005, Phillips 2005)

Table 11: Empirical applications in Macroeconomics and Finance

C Dominance of the deterministic trend over the stochastic trend

In order to explain the dominance of the deterministic trend over the stochastic trend we may study the sums of a UR with drift. Assume x_t is generated by DGP 7; $x_t = \mu_x + x_{t-1} + u_{xt}$. By solving recursively this equation we get:

$$x_t = \underbrace{X_0}_1 + \underbrace{\mu_x t}_2 + \underbrace{\xi_{xt-1}}_3$$

where the first term (1) is an initial condition, the second term (2) represents the deterministic trend, and the third term (3) accounts for the stochastic trend; note that $\xi_{xt} = \sum_{i=1}^t u_{xt-i}$. Then (all sums run from 1 to T),

$$\sum x_{t-1} = \underbrace{X_0 T}_{O_p(T)} + \underbrace{\mu_x \sum (t-1)}_{O_p(T^2)} + \underbrace{\sum \xi_{xt-1}}_{O_p(T^{\frac{3}{2}})} \quad (6)$$

$$\begin{aligned} \sum x_{t-1}^2 &= X_0^2 T + \underbrace{\mu_x^2 \sum (t-1)^2}_{O_p(T^3)} + \underbrace{\sum \xi_{xt-1}^2}_{O_p(T^2)} + 2X_0 \mu_x \sum (t-1) \\ &\quad + 2X_0 \sum \xi_{xt-1} + \underbrace{\sum (t-1)\xi_{xt-1}}_{O_p(T^{\frac{5}{2}})} \end{aligned} \quad (7)$$

The orders in probability of the underbraced sums can be found in Hamilton (1994, chapters 16 and 17). Note that the leading term in expressions (6) and (7) is always the deterministic component (T^2 in $\sum x_{t-1}$ and T^3 in $\sum x_{t-1}^2$). This is why it is said that the time trend asymptotically dominates the stochastic components.

D Asymptotics of LS estimates of specification 2

We present a guide as to how to obtain the order in probability of the estimates and their associated t-ratios using LS where the variables y and x are generated by DGP 4 and specification 2 is estimated. Proof of such was provided with the aid of *Mathematica*

4.1 software. We use the classical LS. Let $\hat{\Theta} = (\hat{\alpha}, \hat{\beta}, \hat{\delta})'$:

$$\begin{aligned}\hat{\Theta} &= (X'X)^{-1} X'Y \\ \text{Var}(\hat{\Theta}) &= \hat{\sigma}^2 (X'X)^{-1} \\ t_{\hat{\alpha}} &= \frac{\hat{\alpha}}{\sqrt{\hat{\sigma}_{\hat{\alpha}}^2}} \\ t_{\hat{\beta}} &= \frac{\hat{\beta}}{\sqrt{\hat{\sigma}_{\hat{\beta}}^2}} \\ t_{\hat{\delta}} &= \frac{\hat{\delta}}{\sqrt{\hat{\sigma}_{\hat{\delta}}^2}}\end{aligned}$$

where,

$$X'X = \begin{bmatrix} T & \sum x_t & \sum t \\ \sum x_t & \sum x_t^2 & \sum x_t t \\ \sum t & \sum x_t t & \sum t^2 \end{bmatrix}; \quad Z'Y = \begin{bmatrix} \sum y_t \\ \sum x_t y_t \\ \sum y_t t \end{bmatrix};$$

and,

$$\begin{aligned}\hat{\sigma}^2 &= T^{-1} \left[\sum y_t^2 + \hat{\alpha}^2 T + \hat{\beta}^2 \sum x_t^2 + \hat{\delta}^2 \sum t^2 - 2\hat{\alpha} \sum y_t - 2\hat{\beta} \sum x_t y_t \right. \\ &\quad \left. - 2\hat{\delta} \sum y_t t + 2\hat{\alpha}\hat{\beta} \sum x_t + 2\hat{\alpha}\hat{\delta} \sum t + 2\hat{\beta}\hat{\delta} \sum x_t t \right]\end{aligned}$$

We shall now describe the process involved in establishing the aforementioned proof. $\hat{\alpha}$, $\hat{\beta}$, $\hat{\delta}$ and their corresponding t-ratios are functions of the following expressions (unless indicated otherwise, all sums run from $t = 1$ to T). Let $w = y, x$:

$$\begin{aligned}
\sum w_t &= \mu_w T + \beta_w \sum t + \underbrace{\sum u_{wt}}_{O_p(T^{\frac{1}{2}})} \\
\sum w_t^2 &= \mu_w^2 T + \beta_w^2 \sum t^2 + \underbrace{\sum u_{wt}^2}_{O_p(T)} + 2\mu_w \beta_w \sum t \\
&\quad + 2\mu_w \sum u_{wt} + 2\beta_w \underbrace{\sum u_{wt}t}_{O_p(T^{\frac{3}{2}})} \\
\sum w_t t &= \mu_w \sum t + \beta_w^2 \sum t^2 + \sum u_{wt}t \\
\sum x_t y_t &= \mu_y \sum x_t + \beta_y \sum x_t t + \sum x_t u_{yt} \\
\sum x_t u_{yt} &= \mu_x \sum u_{yt} + \beta_x \sum u_{yt}t + \underbrace{\sum u_{xt}u_{yt}}_{O_p(T^{\frac{1}{2}})}
\end{aligned}$$

where,

$$\begin{aligned}
\sum t &= \frac{1}{2} (T^2 + T) \\
\sum t^2 &= \frac{1}{6} (2T^3 + 3T^2 + T)
\end{aligned}$$

The orders in convergence of the underbraced expressions can be found in Hamilton (1994) ch. 16. We can fill the previously-cited matrices and then compute the LS parameter estimates and the t-statistic associated with each one. The asymptotics are computed by the program (available at the following URL: <http://www.ventosa-santaularia.com/JPSappD.pdf>).

Notes

¹That said, it should be acknowledged that neither unit roots nor deterministic trends are able to model satisfactorily most series; they are what Phillips (2001) labeled ‘heroically naive’ concepts.

²Particularly noteworthy among the various spin-offs of the aforementioned studies are the Error-Correction-Model (first proposed by Sargan (1964) as a link between static equilibrium economic theory and dynamic empirical models, and further developed by Hendry and Anderson (1977), Davidson, Hendry, Srba, and Yeo (1978), Hendry and Mizon (1978) and Hendry, Pagan, and Sargan (1984).) and the entire theory of cointegration (First proposed by Granger (1981), Granger and Weiss (1983) and Engle and Granger (1987). The linking of Cointegration with Error-Correction-Models can also be found in these articles). Spurious regression has contributed to the general improvement in the level of empirical work.

³A variant of this example, used by the authors to demonstrate the danger of nonsense correlation was taken seriously 100 years ago, by Jevons (1884).

⁴ $I(d)$ notation, for d an integer, refers to the number of differences to be performed so the variable becomes stationary.

⁵We could technically refer to this as Type II spurious regression. This distinction was first proposed by Chiarella and Gao (2002) as an analogy of the classical type I and type II errors in statistics: (1) Type I spurious regression: rejection of a true relationship, and, (2) Type II spurious regression: acceptance of a false relationship. Most of the literature refers exclusively to Type II spurious regression, which we will simply refer to as spurious regression.

⁶Notice that DGP 6 is a special case of DGP 10; nevertheless, both shall be treated as if they were different, given that each has been considered independently in the literature.

⁷That is, it returns to its equilibrium or long-run behavior after any random shock.

⁸As noted earlier, Pearson (1897) and Yule (1897) became aware of several problems concerning the interpretation of the correlation coefficient at the end of the nineteenth century, forging by the way the concept of spurious correlation. The debate, in that time, was greatly related with the interpretation of correlation as causality although the seeds of the modern interpretation that prevails in econometrics are also present; further details can be obtained from Aldrich's (1995) historical review.

⁹Valkanov suggests a $T^{\frac{1}{2}}$ normalization of the t-ratio and the simulation of the distribution in the context of Long-Horizon regressions; Sun proposes a consistent t-statistic with a nuisance-free asymptotic distribution, and; Kim, Lee and Newbold precisely emphasize the fact that, even if the t-ratios of their regression do not diverge, inference can not be drawn because of the nuisance parameters

¹⁰Applying the logarithmic transformation in econometrics is typical when the practitioner wants either to homogenize the variance or to directly obtain estimates of average elasticity amongst variables.

¹¹Entorf's results apply to fixed-effects models where N (the dimension of the cross section) is treated as fixed. The phenomenon of spurious regression in panel data under more general conditions has been further developed; Phillips and Moon (1999) provided a regression limit theory for non stationary panel data with large numbers of cross section and time series. Kao (1999) studied the Least-Squares Dummy Variable estimator (LSDV) where the spurious regression phenomenon is still present for independent non-stationary variables generated by DGPs (6) and (7).

¹²Bhattacharya, Gupta, and Waymire (1983) proved that the R/S test yields spurious inference concerning long memory when the data have a deterministic trend mechanism. See section 4.

¹³Here the underlying theory is an invariance principle (see Davydov, 1970, Theorem 2, and Sowell, 1990, Theorem 1).

¹⁴The variables may be integrated of order $d \in (\frac{1}{2}, \frac{3}{2})$ which generalizes Phillips's (1986) results.

¹⁵The same argument was advanced by Marmol (1998). It should be mentioned that the alternative strategy, 'detrending' (removing the trend component in the data by running a regression on time) was also known to provide spurious results (Nelson and Kang 1981, Durlauf and Phillips 1988).

¹⁶See Dickey and Pantula (1987), for example.

¹⁷See also Cappuccio and Lubian (1997).

¹⁸See also Mikosch and Stărică (2004).

¹⁹DGP innovations may be drawn from distributions other than normal (Cauchy, Exponential, Laplace and so on), although these remain i.i.d. and independent of each other.

²⁰Furman and Zitakis (2009a) p.7 suggest that "the validity of the CAPM [**C**apital **A**sset **P**rice **M**odel] is closely related to the linearity of the regression function of the return on asset i given the return on the market portfolio of all assets". Nonlinearities such as mean or variance shifts may induce the spurious regression phenomenon; this implies that the CAPM beta regression estimations should be performed cautiously.

²¹The non-normal behavior of financial data has been widely documented [see Fama (1965)].

²²Obviously not every combination was carried out by Noriega and Ventosa-Santaulària (2007); besides those presented previously, the following combinations were made by Hassler (1996) and Hassler (2003): DGP 6-DGP 2, DGP 2-DGP 3, DGP 3-DGP 2, and , DGP 3-DGP 3.

²³See Newey and West (1987).

²⁴These residuals are neither normal nor serially independent or homoskedastic; hence, the statistical tests provide appropriate inference by rejecting the null.

²⁵Of course, there is an important exception: if both variables are independent $I(1)$ processes, the Engle and Granger test tends to accept the null of no-cointegration.

²⁶The t-statistic may diverge toward $+\infty$ or $-\infty$; the sign depends on the unknown values of the DGPs; Monte Carlo evidence is presented to assert that in many cases, such sign is negative.

²⁷That is, using the variables in levels, without doing any kind of temporal aggregation.

²⁸DF-type tests (i) over-accept the null hypothesis of unit root when there is a trend/level break in the—trend—stationary process (Perron 1989, Perron 1990, Montañés and Reyes 1998, Montañés and Reyes 1999,

Leybourne and Newbold 2000, Sen 2001, Sen 2003, Kim, Leybourne, and Newbold 2004), and; (ii) over-reject the null hypothesis when there is a trend/level/variance break in the unit root process (Hamori and Tokihisa 1997, Leybourne, C. Mills, and Newbold 1998, Kim, Leybourne, and Newbold 2002, Sen 2008).